[11-01-09-T11]

Answers to 08-02-10-MT-11 [2.6] Page 93

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[1.1] $-\frac{\sqrt{3}}{2}$

[1.2] 1

[1.3] $\frac{\sqrt{3}}{2}$

[2.1] IV

[2.2] IV

[3.1] $\sin(3\pi + \alpha) = \sin(2\pi + \pi + \alpha) = \sin(\pi + \alpha) = -\sin\alpha = -k.$

[3.2] $\cos(-\alpha) = \cos\alpha = \pm\sqrt{1-\sin^2\alpha} = \pm\sqrt{1-k^2} = -\sqrt{1-k^2}$, $\because \alpha$ in II

[3.3] $\cos(\pi - \alpha) = -\cos\alpha = \pm\sqrt{1-\sin^2\alpha} = \pm\sqrt{1-k^2} = \sqrt{1-k^2}$, $\because \pi - \alpha$ in II

[3.4] $\tan(\frac{\pi}{2} + \alpha) = \frac{1}{\tan\alpha} = \frac{\cos\alpha}{\sin\alpha} = \frac{-\sqrt{1-k^2}}{k}$

[4.1] $-\frac{7}{2}$

[4.2] $\frac{3}{2}$

[5] $\cos\alpha = \sqrt{1-\frac{4}{10}} = \sqrt{\frac{96}{100}} = \frac{2\sqrt{6}}{5}$, $\sin\beta = \sqrt{1-\frac{36}{100}} = \frac{4}{5}$. So,

[5.1] $\cos(\alpha - \beta) = \frac{2\sqrt{6}}{5}\left(\frac{6}{10}\right) + \left(\frac{2}{10}\right)\left(\frac{4}{5}\right) = \frac{4}{25} + \frac{6\sqrt{6}}{25}$

[5.2] $\frac{48\sqrt{6}-7}{125}$

[6.1] $\frac{4\pi}{3}$ or $\frac{5\pi}{3}$

[6.2] $\frac{\pi}{2}$

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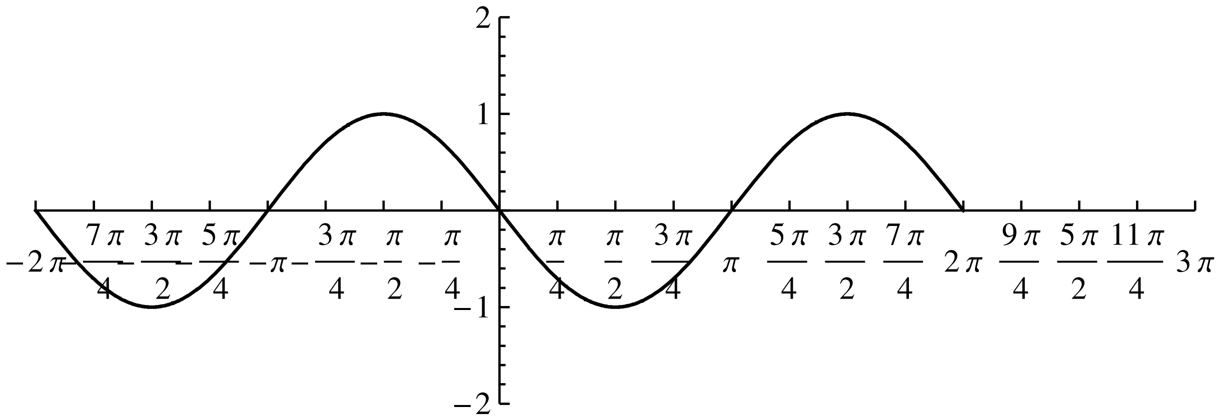
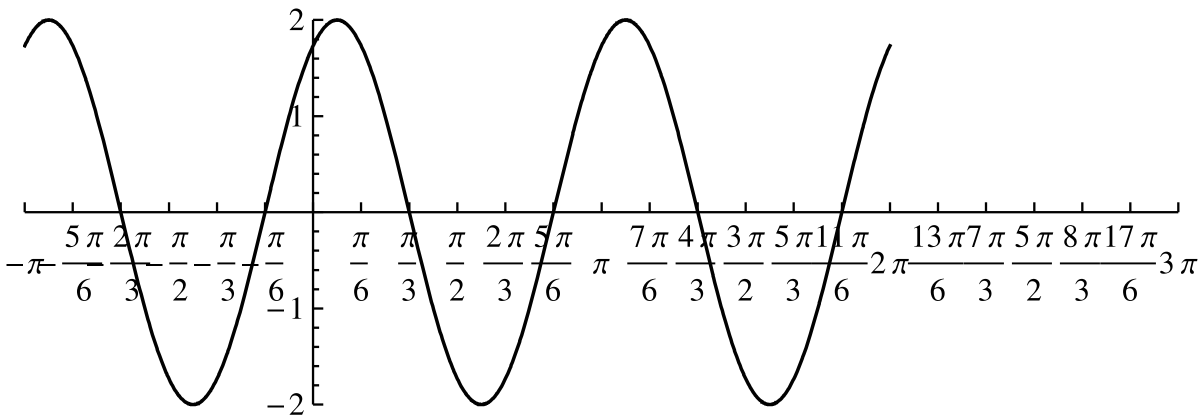
[7.1]



[7.2]



[1.1] Proof: $\frac{l}{2\pi r} = \frac{\theta}{2\pi} \Longleftrightarrow l = r\theta$ □

[1.2] Proof: $\frac{S}{\pi r^2} = \frac{\theta}{2\pi} \Longleftrightarrow S = \frac{1}{2} r^2 \theta$ □

[2.1]

Proof:

LHS

$$= \frac{1+\cos\theta}{1-\sin\theta} - \frac{1-\cos\theta}{1+\sin\theta}$$

$$= \frac{1+\sin\theta+\cos\theta+\cos\theta\sin\theta-1+\cos\theta+\sin\theta-\sin\theta\cos\theta}{1-\sin^2\theta}$$

$$= \frac{2\cos\theta+2\sin\theta}{\cos^2\theta}$$

$$= \frac{2\left(\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)}{\frac{\cos^2\theta}{\cos\theta}}$$

$$= \frac{2(1+\tan\theta)}{\cos\theta}$$

= RHS

□

[2.2]

Proof:

LHS

$= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2\,(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{2}{\sin\theta}$

= RHS

□

[3] Note: since $2-\sqrt{3} > 0$, θ must be in Quad I or IV.

$\sec^2\theta$

$= \tan^2\theta + 1$

$= \left(2-\sqrt{3}\right)^2 + 1$

$= 4 - 4\sqrt{3} + 3 + 1$

$= 2\left(4-\sqrt{3}\right)$

$= 2\left(1-\sqrt{3}\right)^2$

So that,

$\sec\theta$

$= \pm\sqrt{2}\left(1-\sqrt{3}\right)$

$= \pm\left(\sqrt{2}-\sqrt{6}\right)$

Then,

$\cos\theta = \frac{\pm 1}{\sqrt{2}-\sqrt{6}} \iff \boldsymbol{\cos\theta = \pm\frac{\sqrt{2}+\sqrt{6}}{4}}$

For the sine,

$\sin^2\theta$

$= 1 - \cos^2\theta$

$= 1 - \frac{2+6+2\sqrt{12}}{16}$

$= 1 - \frac{8+4\sqrt{3}}{16}$

$= \frac{8-4\sqrt{3}}{16}$

$= \frac{4-2\sqrt{3}}{8}$

$= \frac{4-2\sqrt{3}}{8}$

$= \frac{\left(1-\sqrt{3}\right)^2}{8}$

$\Longleftrightarrow \sin\theta = \pm\frac{1-\sqrt{3}}{2\sqrt{2}} = \pm\frac{\sqrt{2}-\sqrt{6}}{4}$

$\therefore\ \cos\theta = \frac{\sqrt{2}+\sqrt{6}}{4},\ \sin\theta = \frac{-\sqrt{2}+\sqrt{6}}{4}$ or $\cos\theta = \frac{-\sqrt{2}-\sqrt{6}}{6},\ \sin\theta = \frac{-\sqrt{2}+\sqrt{6}}{4}$

You may want to ask about this in class.

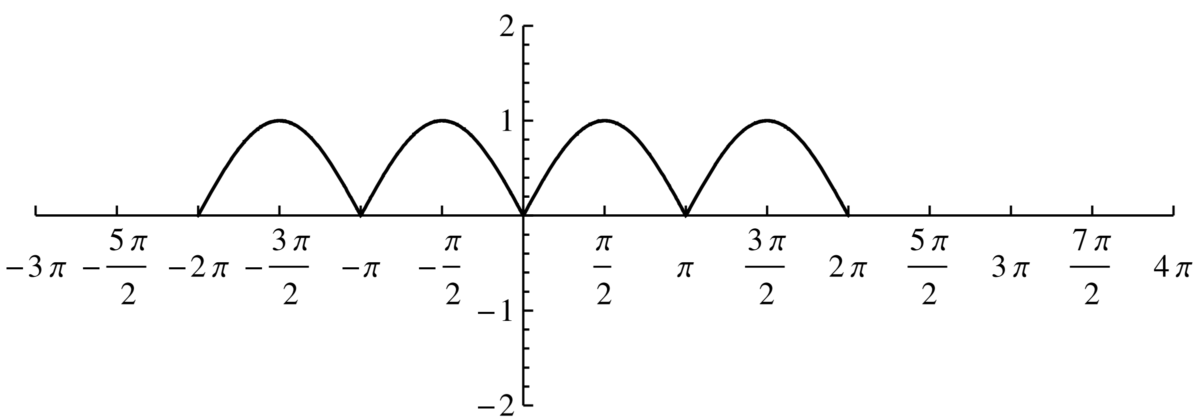
[4.1] $T = \frac{\pi}{\frac{1}{3}} = 3\pi$

[4.2] $T = \frac{2\pi}{2} = \pi$

[5.1] Need these

[5.2] Need these

[6]



[7]

$\sin\theta = 2\sin^2\theta \Longleftrightarrow 2\sin^2\theta - \sin\theta = 0 \Longleftrightarrow \sin\theta(2\sin\theta - 1) = 0.$

Then,

$\text{Sin}\,\theta = 0$ or $2\sin\theta - 1 = 0$

$\therefore\ \theta = 0$ or $\theta = \frac{\pi}{6}$ or $\sin\theta = \frac{5\pi}{6}$

[8.1]

$$\begin{aligned}
&\sin 75^\circ + \sin 15^\circ \\
&= 2\sin\frac{75^\circ + 15^\circ}{2}\cos\frac{75^\circ - 15^\circ}{2} \\
&= 2\sin(45^\circ)\cos(30^\circ) \\
&= 2\left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\right) \\
&= \frac{\sqrt{6}}{2}
\end{aligned}$$

[8.2]

$$\begin{aligned}
&\sin 75^\circ - \sin 15^\circ \\
&= 2\cos\frac{75^\circ + 15^\circ}{2}\sin\frac{75^\circ - 15^\circ}{2} \\
&= 2\cos(45^\circ)\sin(30^\circ) \\
&= 2\left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right) \\
&= \frac{\sqrt{2}}{2}
\end{aligned}$$